Presentation Given At Hebrew University Conference, October 23rd, 2018

Musical and Mathematical Notation - Improving Performance by Moving Through Imaginary Spaces

Summary:

Current research suggests that when we use tools, we extend our body schema. By improving the accuracy of the body schema when we are not holding a tool, we can frequently improve our functionality. If the tool extends our body schema and we improve it, the improvement may extend into the use of the tool. The research also reveals that under the right circumstances, the body schema can be altered to make a person believe that their body is in a configuration which is physically impossible. Music notation can be show to act as a tool that extends the body schema into a musical "space" that transcends the purely physical, for instance in our ability to conceive of a piece of music outside of a time-bound aural understanding of it. By improving the body schema, we improve the use of the notation as tool, and so improve aspects of our musicianship as it applies to the use of the "musical space." Math also makes use of a notation system which serves similarly as a powerful tool, and which may be shown to extend the body schema into configurations that do not correspond with the physical, a "mathematical space." If this is the case, we can suggest that improving the use of the body schema as it is extended by math notation will improve the use that notation and therefore improve the understanding of and performance of mathematics.

Part One: The Body Schema, the Use of Tools, and Music Notation

My name is Adam Cole. I design music curricula and have spent a lot of time developing approaches to make learning efficient and effective. I also have an interest in mathematics and have spent a considerable amount of time exploring my own improvement in mathematical acuity over the years.

I'm also a Guild Certified Feldenkrais Practitioner and have researched how our work impacts our ability to learn across the spectrum of human activity. Today I'm going to make the

argument that when we are doing math we are actually moving through an imaginary space, and that if we can improve our movement through that space, we can subsequently improve our learning ability in it. The means of improving our movement through the so-called math space involves forging a more meaningful connection to a math notation system, and the Feldenkrais Method is ideal for this purpose.

The Feldenkrais Method enables people to make remarkable changes to themselves. It has been difficult historically for us as practitioners to fully explain what is happening in terms that are acceptable to those with backgrounds in science who have not experienced the work. Practitioners such as Isabelle Ginot have used recent terminology of "body image" and "body schema" to describe what happens in a Feldenkrais lesson, and I think it's helpful to describe the work in those terms today.

The model of these two distinct body representations assigns to the *body image* the cognitive representation of the body based on stored knowledge and experience, and suggests that it takes precedence in perceptual judgements. Meanwhile, the *body schema* is dependent on continual input from the proprioceptors, largely operates below consciousness, and is more important in the area of body movements. (Proske and Gandevia, 2012, p. 1666) Our ability to abstract a movement from the environmental conditions under which we typically do them and so determine new control strategies "is made possible by a body schema that allows formulating real and imagined actions in the same format." (Morasso et al, 2015, p. 5)

By making use of the language of body image and body schema, we may describe the process of a Feldenkrais lesson in somewhat more precise terms:

1. The practitioner mobilizes the person's unconscious body schema through verbal instructions or physical manipulation

2. Referring to the client's conscious body image, the practitioner directs the client's attention to sensations and to options for action which suggest new ways to represent and replicate their movement and position. In other words, the client's work in this attentive state results in the revision of their body schema.

3. The client discovers that, as a result of their enhanced body-schema and the subsequent ability to do what was previously difficult or impossible, that their body image, perceptual, conceptual and emotional, may have changed. (Bardet and Ginot, 2012, p.9)

We take a risk when we reduce our work to scientific terms that we will lose the overreaching thrust of the work. We are "humans," "people," not body schemae and body images. We must always return to this understanding if the work is to have meaning and be successful.

Yet just as differentiating a function like reaching, looking at small pieces of it devoid of any goal or desire, gives us a profound understanding of it that only becomes apparent after we re-integrate it. I believe that if we take this small view of our work in the Method, if we take this scientific idea of the body schema out of the big picture for a moment and examine it, we will learn something vital about the totality of the work. We will learn how it helps us to improve ourselves as humans. We will learn how it helps us to improve humanity.

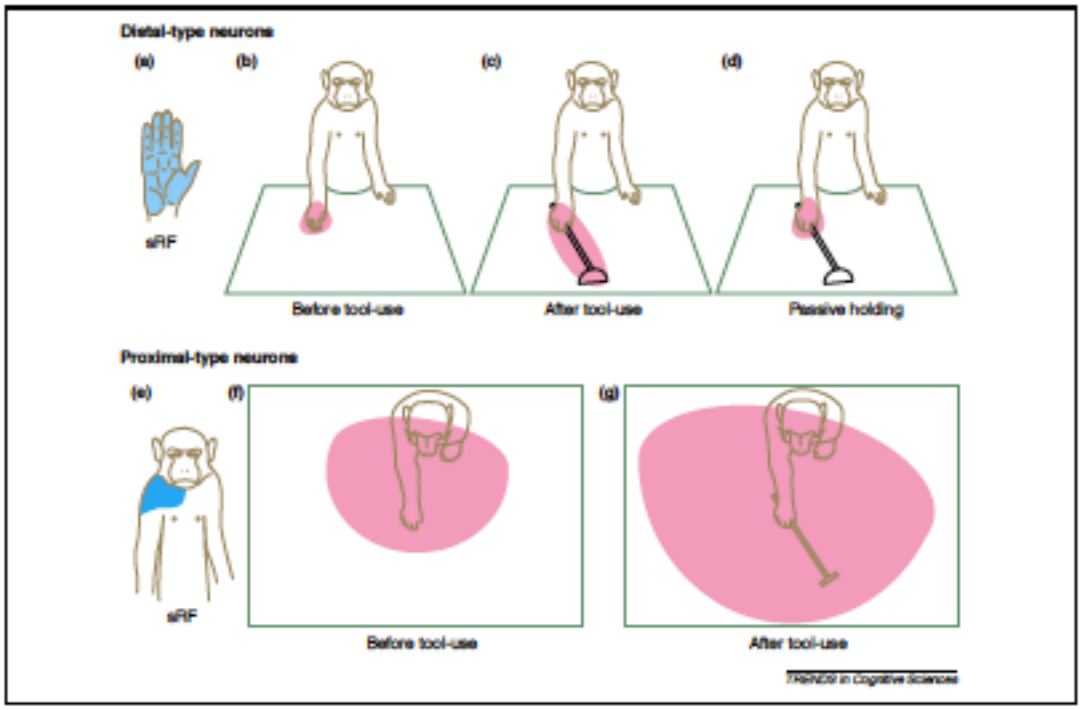
The body schema is a "neural representation of the body that involves motor and posture control," and which is constantly being updated by input from the nervous system. It makes use of our proprioceptive sense, located among other places in parts of the musculature.

Among other things, the body schema lets us know in a dark room where our limbs are in space. (Naito, Moriya, Amenmiya, 2016) Our ability to update the body schema allows us to improve in our ability to function in the physical world under changing conditions. Yet even though the body schema is constantly updating, it can be inconveniently stable, creating phantom limb sensations in body parts that have been amputated. Clearly, the input from the nervous system must be received in a way that is comprehensible to the body schema for it to remain accurate. (Proske and Gandevia, 2012)

Because of its dependence on visual stimula, the body schema can actually be tricked to make a person believe that they are in a configuration which is physically impossible. In one experiment, a subject was asked to close their eyes and touch their nose. Their muscles were vibrated in such a way that they had the sensation their arm was opening to extension, even though the arm remained contracted. Because of the contact between the nose and the hand, the subject believed that as their arm extended, their nose grew to a ridiculous length. (Lackner, 1988)

The fact that we can alter our body schemas to enter into imaginary configurations is vitally important to our topic today. It suggests that our work as Feldenkrais practitioners to engage with inaccurate body images and correct them is supported by research. But it also has implications for our use of tools.

According to a 2004 review of the literature by Maravita and Irki, the research suggests that our use of a tool, even one as simple as a stick, changes the way we think about our body and how to use it. “Recent neurophysiological, psychological and neuropsychological research suggests that this extended motor capability is followed by changes in specific neural networks that hold an updated map of body shape and posture (the putative ‘Body Schema’ of classical neurology).” (Maravita and Iriki, 2004) The illustration below shows the measured change in the visual receptive field (the pink area) of a Japanese macaque before and after using a tool.



The tool is felt as an extension of the body itself, as a living part of it. For instance, researchers discovered that people who are using two long sticks that are crossed to accomplish a task experience the same confusion as if their hands are the things that are crossed. (ibid) There are even instances where subjects who have a prosthetic arm where their regular arm should be demonstrate great alarm when the hand is struck with a hammer, even if their own hand is merely hidden behind a screen!

https://www.youtube.com/watch?v=sxwn1w7MJvk

If we improve the accuracy of our body schema, we can frequently improve the use of our body. As we age, many of our difficulties come about from a use of an incomplete picture of ourselves, for instance when we stabilize ourselves with muscles not meant for the task because we are not internally connected to the stabilizing characteristics of the skeleton. Updating the body schema to alter our self-image for better functionality is one of the working principles of the Feldenkrais Method.

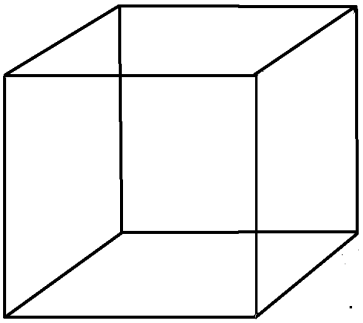
If we are better able to use our body, it follows that we will also be better able to use a tool. Another way of saying this is that our extended body schema will benefit from improvement the same way our normal body schema would. We can improving our reach and power with a stick only if we can make full use of the extended body schema.

A musical instrument is a tool that extends our reach as well. This is especially clear with a stringed instrument like a violin, where the bow as the "voice" of the instrument provides a particular contact with the strings, an accuracy and delicacy, that the human body itself can not produce. The body schema is expanded to encompass the expressive "reach" of the bow.

A piano, similar. Each fingertip is extended by the mechanism of the keys to manipulate up to 10 hammers at a time, each with its own expressive reach. But what is this "expressive reach?" What is the expanded space into which the tool allows us to go?

When I play a key on the piano, the mechanism translates my movement into a certain sound, and the better the piano, the more accurately it translates my movement. If I argue that the music is a result of my physical interaction with the keys, then the "extended space" would simply be a matter of speed or sensitivity that would be accessible no other way than through the instrument. Yet this expressive sound is not the music we are hearing.

The movements we use to actually make a melody really do not resemble our image of how the music moves. My pianist hands move to the right, and we imagine that the music is rising up to the sky. My vocal chords relax while I sing and we hear the music descend.



In fact there are a whole series of "movements" which are not physical, but only musical in nature. A melody can't really be said to move at all, to "rise" or "fall," but we can make physical gestures that approximate the way we experience it. Conductors use tried and true movements, passed through generations, to communicate the music to a chorus.

Yes, the movement of the body is translated into sound, and that sound may be musical. Yet most of us do not hear only the sound, any more than most of us can look at this picture of a Necker Cube and see a hexagon with lines inside. We ascribe meaning to that sound, imagine that it has a shape,

some kind of form, some kind of movement.

Feldenkrais proclaimed that “movement is life.” Among other things, he meant that it serves as our first teacher for understanding how to survive and make sense of our world. Long after we have learned these first movement lessons, we still tend to translate things that do not really move into movement as a way of understanding them, making them clearer: music, language, diagrams of building instructions.

And so we conceive of a disembodied space called the “music space” in which the music moves. Our body, with the extension of the instrument, can enter a space we can’t “go” physically, but can musically. We can represent it physically, like this:

https://www.youtube.com/watch?v=SMMpKYd63Gw

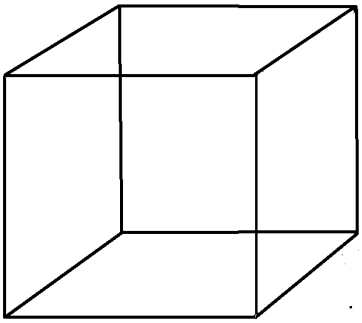
Or this

https://www.facebook.com/Geniusengin/videos/2629463390412507/

Or this.

https://www.youtube.com/watch?v=t18UdM67Plw[1]

And yet the result is a shadow, if you will, of the music, the way the shadow or drawing of a cube gives an impression of its model but does not share its actual properties of only possessing right angles.

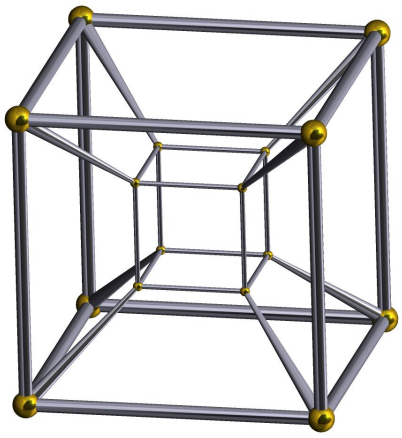


It might be argued that, if there is a one-to-one correspondence between the music we hear and a performer’s physical space and gesture, then the music can be reduced to the physical act of its creation. If this is true, then we would not be able to claim that the “musical space” is actually a non-physical one. It would simply be a physical space that has been translated into

[1] The last of these videos depict students of the Dalcroze Approach, a somatically based medium of music understanding. They are engaged in something called *Animee Plastique*, bodily representations of musical sound.

sound.

And yet we can conceive of music outside of the bounds of its performance, the physical act of its creation, not limited to the moment-by-moment apprehension of sounds as our way of taking it in. If we were limited to a series of infinite moments, we would not be able to understand it, any more than a person with no ability to connect past, present and future moments could understand what was happening to them if they were sliding down a hill. Any representation of music as a series of dots or movements or pictures is only a shadow of a much more complicated experience that involves the integration of time with physical action.



Music, as opposed to a series of sounds, requires that the listener relate what they are presently hearing with what they have heard before. Repeated hearings of a piece of music allow listeners to relate what they are hearing with what they know is coming. This temporal element of music, the interrelationships between different moments in time in a medium bound by time, is what moves the music "space" into imaginary realms that exist only in our minds.

Let's make the analogy more explicit: The image of the Necker Cube gives us a two-dimensional representation of one possible view of the cube. It can't claim to be the cube, because it does not contain all right angles from each vertex. The more images of the cube we show, the better of an idea we can form of the cube's actual nature, but it won't be a real substitution for holding the cube itself in our hands, which would give us the whole cube at once.

A physical model of the hypercube above is a three-dimensional image of one possible view of this four-dimensional shape. It can't claim to be the actual hypercube, because it does not contain all right angles from each vertex, some of which would go into a fourth dimension. However, we can create as many models as we like that depict variations of the hypercube, different "views" of it, and so be able to form in our minds an idea of what that cube would actually be like.

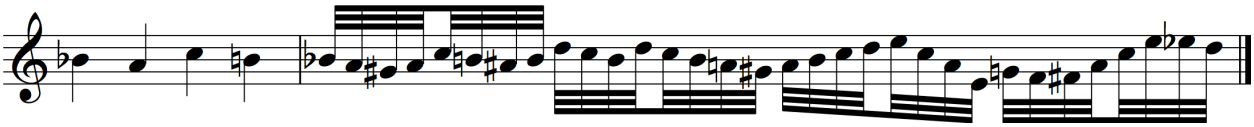
This is the only way we can comprehend the entirety of the hypercube. We connect as many different images or models of it as we can in our imagination, each one flawed in some way or other, and so determine the nature of the object, like the story of the six blind men and the elephant. Our minds are now exploring a space we can't enter.

Now imagine a physical representation of a piece of music such as the videos above. Obviously we must hear (and perhaps see) the entire piece of music to experience and understand it. Yet we can only experience its unfolding over time…we can't listen to the whole thing in one instant, and we certainly can't listen to the ideas that are related to one another at opposite ends of the piece, beginning and end.

All we can do is listen to the whole thing moment by moment, each instance a single slice of the totality of the piece. We integrate the discrete moments in our minds, relating not only consecutive sounds and gestures, but the totality of the moments, A to B, A to C, A to D, and so forth. In order to provide the composition with a meaning for ourselves, as we listen we reconstruct it as a four-dimensional sonic object in our imaginations, because this is the only way we can comprehend the entirety of the composition at once.

Western classical music notation, a series of graphic instructions for these physical gestures, is one commonly accepted shadow of this music by which we can infer its whole shape. It indicates tempo, rhythm and pitch, as well as articulations and sometimes indications of musical form. Someone who has learned this system can see the relationships of pitches and rhythms to one another outside of the "gravity" of time-dependent perception of the music.[2]

It is interesting to note that, just like the drawing of the cube, and the tesseract, the western notation system is not always a good representation of the music it codifies, nor of the movement it requires of the musician. For instance, four seconds of playing time may take up 1 inch or 6 horizontal inches on the printed page, depending on how many notes must be fit into the measure. In the example below, the first four notes take exactly as long to play as the 32 notes that follow them.



As far as the correspondence between Western Classical notation and human movement, we have a disaster on our hands. In a piano score the music is meant to be read continuously from left to right (until it's time to change lines), with higher notes being closer to the top of the page, and lower notes being closer to the bottom. The pianist must translate the image 90 degrees and play higher notes further to the right, and lower notes further to the left, all the while tracking left to right and scanning independently with their eyes! Certainly this is an evolved system, because no one with any brains would have designed it this way from the outset. It's a wonder anyone learns to play at all.

These are only inconveniences for our minds, though they may be insurmountable for some. Over time a musician learns the idiosyncracies of the notation system and rapidly converts symbol to sound-producing movement, the same way we learn the idiosyncracies of our letter notation (*laugh* and *daughter*). Once learned, music notation can greatly extend the ability of a musician to decode musical information and the movement of the music body to express it.

I believe learning to read music greatly enhanced my musicality in that the tool of the notation expanded what I could conceive and keep in my head at once. I can make sense of the orchestral score below by moving my eyes in the right way and keeping certain information in one pocket of my brain while taking in the rest. Given enough time, I can convert this into sounds on the piano, and some musicians can do that trick instantly.

I propose that the symbols of music, once mastered, act the same way as a tool. A player looks at the notation, and it enacts the physical movements she will need in her mind much faster than she would be capable of executing them in real time. She can conceive of and describe a whole piece in seconds, or at once, when it would require minutes to play.

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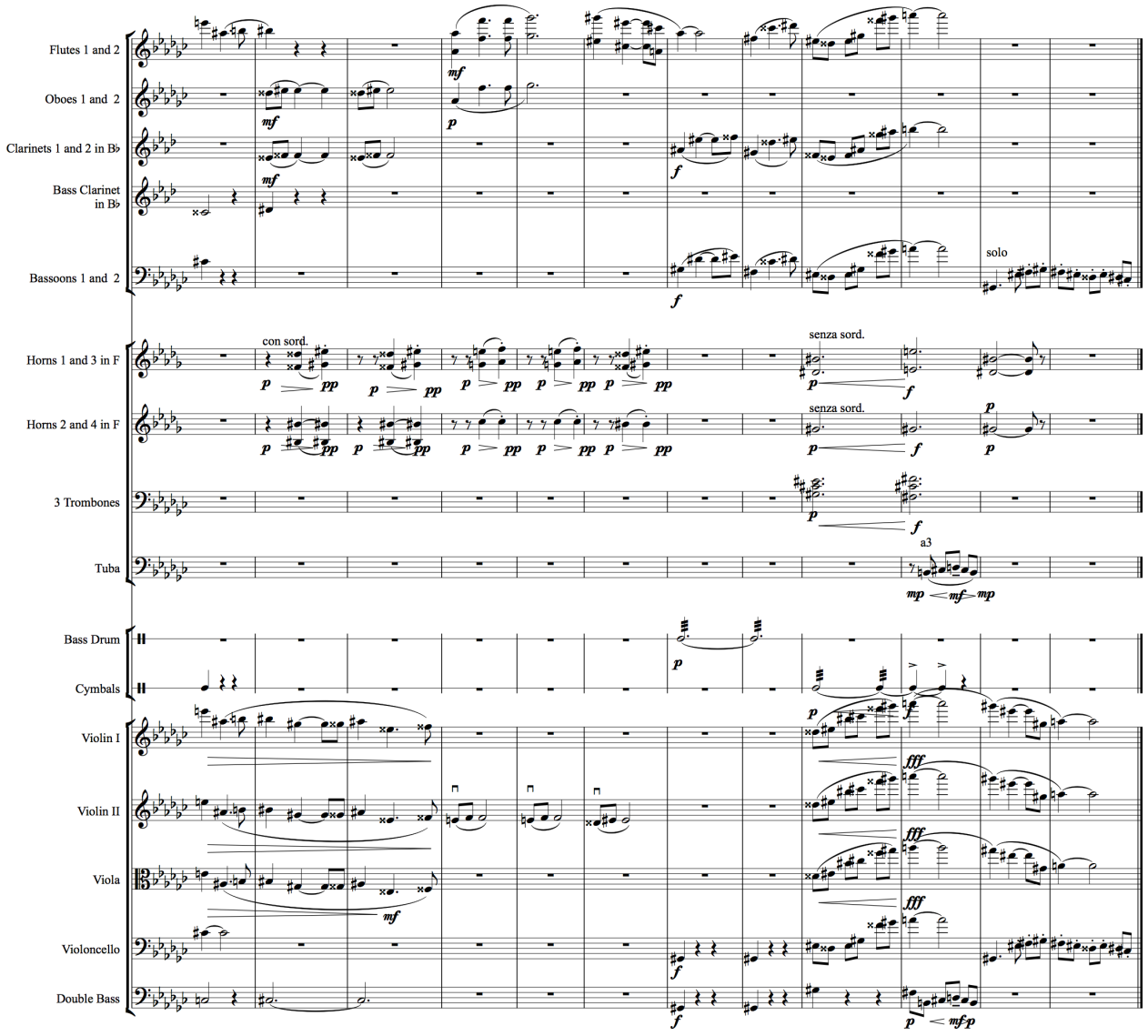
[2] Time enables us to understand music by putting the sounds into their proper context. However, a performance in time also prevents us from lingering on areas that are difficult to understand, or from going backwards and forwards through a piece as we like. Because time is no longer a constraining element in our comprehension, looking at a score is analogous to exploring the movement of our physical bodies free from gravity, able to put ourselves in any orientation that we like with no pull upon our musculature.

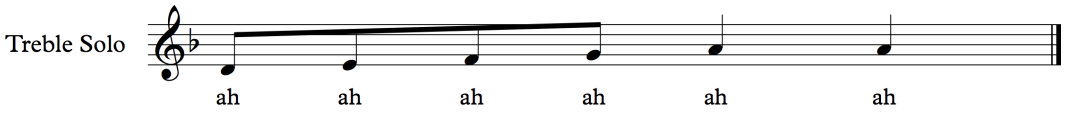


If music notation is a tool, then it extends the body schema into that larger “music space” transcending time, which cannot physically exist but is still measurable and usable. By reading music notation, we extend our body schema into a space that is not dependent on time. We can imagine ourselves as the musician who is simultaneously enacting many or all parts of the music at once.

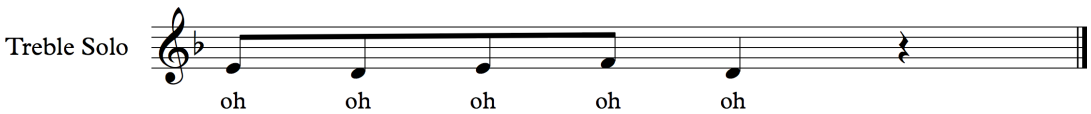
It follows that the use of such a tool, and therefore the musicality it fosters, can be augmented by improving the body schema. For someone simply reading music in a quiet room, that improvement may be made in a very small part of the body, the eyes, hands, perhaps the mouth. The result for the performer might be a more expressive, more knowledgable performance, and the insights they receive from this experience might translate back to an improvement in the use of their physical selves.

Part Two - the author will teach the use of a basic music symbol set to the audience. After learning the symbols, will they experience an enlarging of their sense of self upon seeing the symbols in a new configuration? Will their increased competence with the tool convert a series of unrelated sounds into music for them?

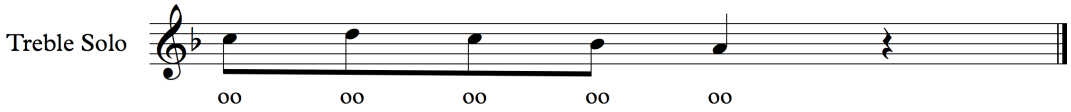
1. Show the audience a quarter note. “Ta.”
2. Show the audience a pair of eighth notes. “Ti ti”
3. Have them call out according to what I point to.
4. Let them assist me in composing a rhythm made up of ti-ti and ta.
5. Put a sequence on the board. #1 “Ti ti ti ti ta ta.” Lead them in reading it. Ask the audience to repeat.
6. Put a sequence on the board. #2 “Ti ti ti ti ta.” Ask the audience to read it.
7. Put a sequence on the board. #3 “Ti ti ti ti ti ti ta.” Ask the audience if they can hear in their heads the sequence before they read it out loud. As the audience to read it.
8. Ask if they know the Israeli National Athem. Have them sing it.
9. Break up the anthem into phrases. Phrase a) <the first five notes of the anthem>. Show



this phrase in notation on a piece of cardboard which a volunteer can hold up. Note that the rhythm is the same as #1.

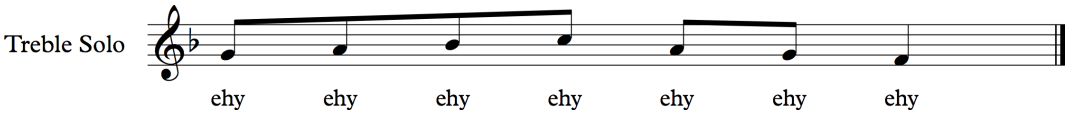


10. Phrase b) <the last five notes of the A-section> Show this phrase in notation on a piece of cardboard which a volunteer can hold up. Note that the rhythm is the same as #2.
11. Phrase c) <notes 5-9 of the B section>. Show this phrase in notation on a piece of



cardboard which a volunteer can hold up.

12. Phrase d) <notes 24-29 of the B-section>. Show this phrase in notation on a piece of cardboard which a volunteer can hold up.



13. Teach the phrases again. Have volunteers hold posters up with the notation (which are also in different colors) in the sequence that they were presented.
14. Move the volunteers around according to the participants instructions. Enable the participants to sing each phrases independent of their position.
15. Restore the participants to their original order. Ask the volunteers to sing the song. Point to the phrases in the order that they appear
16. "Is there a difference in the way you think of the song now? Is your view of the song more complete in that you now know that there are several phrases which appear multiple times in the song, and you now know when they appear? Would it be easier for you to start anywhere in the song, or to teach it to someone else?

Part Three: Mathematics Notation and the Body Schema

A compelling argument can be made that *any* symbol set can serve to expand the body schema if it is connected to some kind of movement. Movement is life, as we have already mentioned, and we abstract concepts from our first lessons in movement. It makes sense that we'd try somehow to make math concepts "move" in order to understand them.

Feldenkrais realized that if he took movement functions like "reaching" and presented them in the context of interesting and practical physical puzzles, we would be much more likely to

engage with the ideas and to integrate them. Similarly, word problems are a way to take an abstract mathematical idea and put it into an interesting mental puzzle. For many, though, certain steps are necessary before they are capable of doing the mathematical "movements" that would engage them in the puzzle.

What we have shown in music is that the "movement" of something does not have to be physical, and yet may be "real" by being consistent, and having consequences. This argument is particularly relevant in the subject of teaching mathematics. I know that in my own imagination I do imagine movement when I think through a proof or an example: symbols rearrange themselves, limits converge, etc.

Mathematics symbols may serve as shorthand notations for actions, some of which are not even possible in the physical world we live in, such as multiplication of extremely large numbers, or movements in 4 or more dimensions. There are those who master the symbols and perhaps get so familiar with them that they see their movements as existing in a "mathematical space."

This math space may have little to do with the physical body, and may even be a powerful escape substitute for it for those that are uncomfortable with their body. Conversely, those who are well connected in their bodies may be unable to connect to the math space, perhaps because they prefer the physical realm with its more limited options. Certainly there are people who are skilled at both, and people who are uncomfortable with both.

The math space can be imagined through the manipulation of symbols which are interconnected by a consistent set of rules, different rules for different symbol sets, but all of which have a kind of "movement" to them, a procedure which is not unlike real movement in that it involves manipulation, orientation and, to a lesser extent, timing. The symbols are certainly manipulated, starting with algebra, and their orientation makes certain solutions more or less apparent (solving for one variable rather than another). The question of timing may be a physical one, in that if a person gets stuck on one part of a mathematical argument, it will be very difficult for them to maintain a grasp of the whole, the same way a piece of music will not be effective if we stop on a single note. There also may be deeper notions of "timing" inherent in mathematics, the visualization of elements that move at the same or different rates (x = y^2)

The math schema has invariable constraints in its procedures { The parentheses rule: 3 + (7 x 2) = 17 } {Algebraic operations: f=ma is equivalent to f/a = m}, as well as constraints which can be set and changed ("Let x be a real number between 3 and w"), and even constraints that can be redefined (the trigonometry of flat surfaces versus the trigonometry of curved surfaces). We might suggest an analogy between the fixed constraints and the physical limits of the human body, such as the degree to which we can extend our arm, and the variable constraints with the places where only the imagination can serve, such as a nose that can grow to a foot in length.

If the math symbol set is seen as a tool, then connecting to the math symbol set and using it effectively would expand the person's body schema into its imaginary realms. The success of this endeavor would require that the person have a body schema sufficient to connect to the math space. As we've already mentioned, there are plenty of people whose understanding of the math space is good but whose body schema is limited and so perhaps only certain parts of the schema are necessary - eyes, mouth, or a substitute like hands if mouth and eyes are damaged.

A good body schema would not be sufficient to master the math space, as the body's connection to the tool is not a given. Furthermore, having the mental capacity to understand the math space would not be sufficient, as the student must practice with the notation for

fluency in order to have a reasonable shot at learning more difficult branches of mathematics. Some people get stuck at algebra, others at calculus, etc. and this may be less about their *capacity* to understand then it is about their current connection between their schema and the tool. Just as it is very difficult to adequately assess the musical acuity of a person born deaf, in order to gauge true mathematical acuity it would be necessary to separate the elements that make comprehension possible from those that make it difficult.

I believe that improving the connection to the math space with the physical body can be done in three ways:

1) By improving *in isolation* the physical functions that are in operation when reading mathematics symbols
   - scanning eye movements and breath for sufficient continuity of information intake, so that large enough concepts can be grasped as a whole;
   - writing or typing or drawing functionality
   - anxiety reactions.
2) By improving the body image to make the comprehension of the math space clearer. Gaining a sense of three-dimensionality, a 3-D internal schema, may enable one to conceive of manipulating symbols that have 3 or more dimensions to them, such as a graph with three axes, or a problem with different classes of variables. Without this mature sense of movement in the body, it may be difficult or impossible for some people to conceive of these concepts in the abstract.
3) By experiencing continuity of a human function like reaching (smoothness, reversibility) in the body, as a means of modeling such continuity in the manipulation of symbols, movement within the "math space."

Each of these ideas suggests improving the body schema first, as a means of improving comfort with the use of the math notation system and, by extension, the ideas of mathematics. It might be possible, however, to do Feldenkrais-type activities with the body as extended into the math space itself. This would mean looking at the body intersecting with the math space as a kind of enhanced body and working with it.

One such exploration might involve the act of writing in the mathematics process. Having established through research that a tool will extend the body schema, we must allow that a pencil is such a tool. If the mathematics notation system is also a tool that represents the math-schema, then the use of the human body in the act of writing the symbols would be an area of intersection between it and the math space.

I can imagine asking a student to write from memory the steps of the proof of the irrationality of the square root of 2, and asking the student to observe where the process of writing appears to be halted or discontinuous. Is the issue a lack of continuity in comprehension of the "movement" of the math space from one step to another? Can an activity be designed that creates an opportunity for the student to write continuously on a more differentiated piece of the proof that contains the difficulty, and will this be superior to simply attempting to explain the concept to them?

These types of teaching are likely already happening with really good math teachers who simply consider this "good teaching." Others are exploring these ideas overtly, though perhaps they lack a comprehensive way of describing them. It's my hope that this presentation clarifies that approach.

Finally, it is possible that this kind of learning could and does go both ways - that learning mathematics in the right way would help someone improve their use of the body schema. To

do this consciously, to teach it, would benefit mathematics as a subject and education in general. From a somatic perspective it would humanize math and humans alike.

Sources and Further Reading

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